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Samuel Ferey and Pierre Dehez



**CORE**

## DISCUSSION PAPER

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## Multiple Causation, Apportionment and the Shapley Value

Samuel Ferey<sup>1</sup> and Pierre Dehez<sup>2</sup>

March 2015

### Abstract

Multiple causation is one of the most intricate issues in contemporary tort law. Sharing a loss suffered by a victim among multiple tortfeasors is indeed difficult and Courts do not always follow clear and consistent principles. Here, we argue that the axiomatic approach provided by the theory of cooperative games can be used to clarify that issue. We have considered the question from a purely game theoretic point of view in Dehez and Ferey (2013). Here we propose to analyze it in a legal perspective. We consider in particular the difficult case of successive causation to which we associate a general class of games called “*sequential liability games*”. We show that our model rationalizes the two-step procedure proposed by the *Restatement Third of Torts*, apportionment by causation and apportionment by responsibility. More precisely, we show that the weighted Shapley value associated to a sequential liability game is the legal counterpart of this two-step procedure.

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## **Outline**

### **1. Introduction**

### **2. Liability games**

2.1 Heuristic presentation

2.2 Liability games

2.3 The core

2.4 The symmetric Shapley value

2.5 The weighted Shapley value

2.6 Beyond three players

### **3. Applying the weighted Shapley value to the Law**

3.1 The Shapley value as a normative tool

3.1.1 The Shapley value properties

3.1.2 Comparing the Shapley value with alternative allocation rules

3.1.3 Incentives and the Shapley value

3.2 The Shapley value as a descriptive tool

3.2.1 Example and cases

3.2.2 A more systematic view: *The Restatement* and the Shapley value

### **4. Concluding remarks**

## **References**

# 1. INTRODUCTION

*"Logic has not always the last word in law"*

Chief Justice of the New Hampshire  
Supreme Court Robert Peaslee<sup>1</sup>

Multiple causation is one of the most intricate issues in contemporary tort law which arises when several tortfeasors cause harm to a victim entitled to recover it: Courts have to apportion damages among them<sup>2</sup>. Many subfields of private law are concerned with apportionment issues: environmental law (several firms poisoning a river), medical malpractices (surgeon aggravating the consequences of a first accident caused by an initial injurer), health litigation (asbestos exposure by several firms through time), antitrust law (dividing the loss suffered by the consumers due to antitrust practices by several firms) *etc.* Many models and theories have been proposed in law<sup>3</sup>, philosophy, economics<sup>4</sup>, psychology<sup>5</sup> to capture the features of legal causation and apportionment issues. These legal debates lead the American Law Institute to promulgate a new *Restatement* dedicated to this issue<sup>6</sup>.

The present paper adds to this literature by developing a game theoretic approach in which damages are monetized and modeled as cooperative games where players are the tortfeasors who jointly created an indivisible economic loss to be paid.<sup>7</sup> Solution concepts are then applied following an axiomatic method. Contrary to law and economics models in the literature, we are more interested in the fairness of the apportionment than in the incentives created by the apportionment rules. Therefore we consider causation from an *ex post* perspective – once the damage occurred – and not from an *ex ante* perspective.<sup>8</sup>

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<sup>1</sup> Peaslee (1934, p.1131).

<sup>2</sup> Four multiple causation issues may be distinguished: successive causation, simultaneous causation, alternative causation and victim's contribution.

<sup>3</sup> See Hart, and Honoré (1985) and Borgo (1979) for a comprehensive analysis of causation related to the theory of law and Coleman (1992) for causation issues related to moral theory. See also Wright (1985a), Keeton (General Editor), Dobbs, Keeton, and Owen (1984) for specific issues of causation in Torts.

<sup>4</sup> In economics, a constant attention has been devoted to this topic. See Landes, and Posner (1980); Rizzo, and Arnold (1980), (1986); Shavell (1983); Kornhauser, and Revesz (1989); Young, Faure, Fenn, and Willis (2007); Parisi, and Singh (2010).

<sup>5</sup> For a psychological approach on causation in law, see Rachlinski (1998) and more generally the literature about the hindsight bias in behavioral law and economics.

<sup>6</sup> See the *Restatement (Third) of Torts: "Apportionment of Liability"* promulgated by the American Law Institute in 1999 and published in 2000, notably "Topic 5: Apportionment of Liability When Damages Can Be Divided By Causation, § 26" ([thereafter](#), the *Restatement*).

<sup>7</sup> For a comprehensive view of the economics of causation, see Ben-Shahar (2000). Surprisingly enough, the theory of cooperative game and its solution concepts have never been elaborated in the law and economics literature to analyze multiple causation issues. To our knowledge, and except for an unpublished paper mentioned by Ben-Shahar (2000), no model of multiple causation cases is available in terms of cooperative game. See also the approach proposed by Braham, and van Hees (2009) which analyzes the concept of "the degrees of causal contribution for actual events" by using power indices.

<sup>8</sup> Our approach is more a retrospective causation perspective rather than a prospective causation perspective. According to Ben-Shahar (2000, p. 647) "Retrospective causation exists if, all else held fixed, but for the action the harmful consequence would not have occurred. Prospective causation exists when an action raises the

Contemplating the debates between legal philosophers and law and economics scholars on causation, the *ex-ante* – *ex-post* distinction could be said to be a *summa divisio*. On the one side, most of legal philosophers interested in corrective justice criticize law and economics findings for its forward-looking oriented theory of causation and prefer developing some *ex post* criteria of causation;<sup>9</sup> on the other side, law and economics scholars, following Coase, try to show that causation is not the keystone of Torts as soon as the legal system seeks to implement optimal incentives. Dealing with causation in law and economics, Cooter wondered "how is legal cause imbedded in formal models? Do the formal models clarify difficult legal issues about causation, as concluded by such writers as Calabresi, Shavell, and Landes and Posner? Is the disappearance of "cause" from the formal models evidence of scientific progress and a reason for celebration, as Russell's views suggest? Or do the formal models obscure legal cause and suppress interesting legal issues, as asserted by critics such as Wright?" (Cooter 1987, p. 523). One of the findings of our approach is to show that economic theory adds also to *ex post* causation theories and apportionment issues. Legal philosophy could learn from economic models of causation in an *ex post* perspective. Such models could then be developed to fill the gap between legal conceptions of causation and law and economic ones. There is another finding of the paper.

In what follows, we distinguish with Posner and Landes (1980) "successive joint tort" and "simultaneous joint tort". Although we cover both cases, we focus successive injury for two reasons: first, these cases have specific mathematical properties; second the counterfactuals needed to implement apportionment rules are more easily knowable than in simultaneous cases. Successive injury occurs when, after an injury caused by a first tortfeasor A to a victim V, the damage is aggravated by tortious acts from a second wrongdoer B, then from a third one C *etc.* A, B, C... are said to be the multiple tortfeasors because they cause *together* the final damage suffered by V. An example from the *Restatement* may illustrate such a case: suppose "A negligently parks his automobile in a dangerous location. B negligently crashes his automobile into A's automobile, damaging it. When B is standing in the road inspecting the damage, B is hit by C, causing personal injury to B. B sues A and C for personal injury and property damage. B's negligent driving and A's negligent parking caused damage to B's automobile. A's negligent parking, B's negligent driving, B's negligent standing in the road, and C's negligent driving caused B's personal injuries." (American Law Institute, 2000, Topic

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probability of the harmful consequence. Thus, the distinguishing factor between the two types of causation is the time perspective of the evaluation. Retrospective causation is backward-looking, answering the counterfactual inquiry of whether the action was a necessary condition for the outcome. Prospective causation, in contrast, is forward-looking, answering the *ex ante* inquiry of whether the action increased the likelihood of injury".

<sup>9</sup> As Cooter says, "Economic models of tort law are based on functional relationships among such variables as the probability of accidents, the harm they cause, and precaution against them. Being mathematical relationships, they are not explicitly causal [...]." (Cooter 1987, p. 523). For a criticism of economic analysis of law related to *ex post* and *ex ante* perspectives on causation, see Wright (1985b) and Coleman (1992). Our approach shows instead that economics has a lot to say on *ex post* causation.

5, §26, *comment c*). How should judges determine the compensation to be paid by each injurer? Should he consider that the car driver A is liable for the entire damage insofar as without his action the damage would have not occurred? Or that each of the tortfeasors is liable for a part of it? Or that one of them is more liable than the other and for which amount? An apportionment rule is needed to correctly share the damage. Such litigations occur as soon as two or more individuals have jointly caused damages and it is easy to think about the different fields of law concerned by this issue: environmental law, nuisance, accidental law, medical malpractices, products liability, insurance law or even antitrust, *etc.*<sup>10</sup>

In our model, an *adjudication* specifies the compensation that each tortfeasor has to pay to the victim. *Adjudications* should be *unobjectionable* (Dehez and Ferey, 2013). There is a *minimum* compensation: each tortfeasors should pay at least the damage that he would have caused *alone*. There is also a *maximum* compensation: no tortfeasor should pay more than the *additional* damage that he has caused. The additional damage is measured by the difference between the total damage and the damage that would have resulted without the participation of that tortfeasor. These two inequalities can actually be found in tort law. Here we go further and extend them from individual tortfeasors to *subsets of tortfeasors*, leading to the following two conditions:

- C1 *the contribution of any subset of tortfeasors should be at least equal to the damage they would have caused without the intervention of the others.*
- C2 *the contribution of any subset of tortfeasors should not exceed the additional damage resulting from their participation.*

To apprehend the notion of unobjectionable adjudications, we construct a game with transferable utility – called *liability game* – whose characteristic function reflects the *potential* damage caused by any subset of tortfeasors while capturing explicitly successive causation. We show that the *core* of a liability game defines the set of *all* unobjectionable adjudications and that the (symmetric) *Shapley value* defines a fair compromise in which tortfeasors differ only in the damage they have caused. A judge may depart from that fair compromise by assigning weights to tortfeasors in order to reflect misconduct or negligence. The resulting *asymmetric* Shapley values define unobjectionable adjudications and, vice versa, weights can be associated to any unobjectionable adjudication.

Both legal practices and economic analysis of law are concerned by our analysis. First, our model provides a characterization of the apportionment rules that could be used by Courts. Second, we show that judicial practices, jurisprudence and legal debates underlie the solution

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<sup>10</sup> Three main approaches are distinguished in law: joint liability, several liability and joint and several liability. In *joint liability*, each tortfeasor is liable for the full amount of the damages, without any claim against the other tortfeasors. In *several liability*, each tortfeasor is only liable for a given share. In *joint and several liability*, each tortfeasor is liable for the total amount of damages but has a claim against the other tortfeasors to get their contribution to damage back. Sharing rules are needed in the last two cases.



concept that we use. For that purpose, we illustrate our model by some Court decisions and by proposals and synthesis provided by the *Restatement*. We show that our approach offers a framework to better understand the two-steps procedure advocated by the *Restatement* based on apportionment by causation and apportionment by responsibility. Cooperative game theory is relevant for law and we aim at making judges and legal practitioners aware of the implicit logic they use to solve such cases. Moreover, discussing apportionment issues on the grounds of an axiomatic method may be useful to achieve greater fairness and greater consistency in adjudication.<sup>11</sup>

The remainder of the paper is organized as follows. In section 2, we define liability games and show that their core defines the set of unobjectionable adjudications. We show that it coincides with the set of weighted Shapley values. Section 3 deals with legal issues. We show how the rule proposed by our cooperative game model enlightens the main legal principles and practices in tort law. We mainly rely on American common law cases on the one hand and on principles and proposals advocated by the *Restatement* on the other. More precisely, we deal with the scope of the Shapley value for the law, normative as well as positive. We show the two-step procedure proposed by the *Restatement* implicitly follows a cooperative game approach and proposes an apportionment method which happens to be equivalent to the core and the weighted Shapley value prescriptions. Section 4 concludes.

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<sup>11</sup> As Coleman (1982, p. 349) asserts "political authority is [necessarily](#) and inevitably coercitive [...], exercising it requires a justification [and therefore] any body of the law must be coherent and consistent". See also Boston (1996, p. 269).

## 2. LIABILITY GAMES

Litigations in multiple causation cases are due to the fact that several tortfeasors have jointly caused damage to a victim. We begin by providing a heuristic presentation of our approach. We then define liability games, with a particular attention to sequential liability, and introduce the concepts of core and Shapley value. Throughout this part, we illustrate our approach by the 2 and 3 player's cases.

### 2.1 Heuristic presentation

To keep things simple, let's consider a situation where two persons are involved in damage whose monetary value  $D$  is known. A Court must allocate  $D$  among the two injurers. This determines the amount each one will be asked to pay. We will consider two cases. The "*simultaneous liability*" where no damage would have resulted if one of the injurer had not been present and the "*sequential liability*" where the damage is successively aggravated.

*Equal division* is the natural allocation in the simultaneous liability case. A Court may however consider that, because of negligence or fault, one injurer should be asked to pay more than the other. Let's identify the injurers as 1 and 2. An *adjudication* is a pair  $(x_1, x_2)$  that specifies an allocation of  $D$  among the two injurers:  $x_1 + x_2 = D$ . A system of non-negative weights  $w = (w_1, w_2)$  summing to one can be associated to a adjudication  $x = (x_1, x_2)$ . They are given by  $x_i = w_i D$  ( $i = 1, 2$ ). They give a measure of the relative responsibility of each injurer.

Sequential liability is more complicated. The first injurer causes an initial damage  $d_1$  that is aggravated by the second injurer who causes an *additional* damages  $d_2$ . The total damage is then given by  $D = d_1 + d_2$ . Imposing to each injurer to pay for "his" additional damage may seem to be, at first sight, a natural solution. It is however not necessarily fair! Indeed, if the first injurer had not been there, no damage would have occurred. Hence, the first injurer should be asked to cover at least the initial damage  $d_1$  and could be asked to pay, on top, part of the additional damage  $d_2$ . A natural solution is to impose to the second injurer to pay *half* of his additional damage. In order to allow for a differential treatment of the injurers, a system of non-negative weights  $w = (w_1, w_2)$  summing to one can be associated to an adjudication  $(x_1, x_2)$  and vice-versa:

$$w_2 = \frac{x_2}{d_2}$$
$$w_1 = 1 - w_2 = \frac{x_1 - d_1}{d_2}$$

or, equivalently:

$$\begin{aligned} x_1 &= d_1 + w_1 d_2 \\ x_2 &= w_2 d_2 = (1 - w_1) d_2 \end{aligned} \tag{1}$$

## 2.2 Liability games

We denote by  $N = \{1, 2, \dots, n\}$  the set of tortfeasors involved in the case. All together, they have caused a final damage  $D$ .<sup>12</sup> For any subset of players, we need to identify the damage that these players would have caused together, without the contribution of the other players. This is the notion of *potential damage* that relies on a counterfactual reasoning. It applies to individual players as well, and the potential damage of the all player set (the "grand" coalition) is the total damage  $D$ . This defines a function  $v$  that associates a real number  $v(S)$  to all possible subsets  $S \subset N$ . The pair  $(N, v)$  is a *cooperative game with side payments* where  $v$  is the *characteristic function* of the game.<sup>13</sup> In a general context,  $v(S)$  is the "worth" of coalition  $S$  and it measures the minimum that coalition  $S$  can ensure by itself if it forms. In our context, these games are called "*liability games*".<sup>14</sup> Liability games are assumed to satisfy a weak form of *superadditivity*:

$$v(S) + v(N \setminus S) \leq v(N) \text{ for all } S \subset N$$

i.e. the sum of the additional damages of a coalition and its complement never exceeds the total damage.

In a simultaneous liability case where each tortious act is a necessary condition to damage, the game is easily identified:  $v(S) = D$  if  $S = N$  and  $v(S) = 0$  for all  $S \neq N$ . This corresponds to the *unanimity game*: no damage occurs once a member of  $N$  is missing.

In the sequential case, players are identified by their position in the sequence. The immediate damage  $d_i \geq 0$  caused by each player is assumed to be known.<sup>15</sup> The corresponding liability game is then entirely defined by the list of immediate damages  $d = (d_1, d_2, \dots, d_n)$ . In the 2-player and 3-player cases, we then successively have:

$$\begin{aligned} v(1) &= d_1 \\ v(2) &= 0 \\ v(1, 2) &= d_1 + d_2 = D \end{aligned}$$

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<sup>12</sup> Players are the injurers and possibly also the victim in which case her indemnity is reduced by the amount she has to pay.

<sup>13</sup> The characteristic function was first introduced by von Neumann and Morgenstern (1944). See Luce, and Raiffa (1957) for an old yet excellent reading, or Maschler, Solan, and Zamir (2013) for a more recent one.

<sup>14</sup> By convention, the empty set is assigned a zero value:  $v(\emptyset) = 0$ .

<sup>15</sup> An immediate damage could be zero. If  $d_i = 0$  for some player  $i < n$ , injurer  $i$  has caused *indirectly* a damage. If the victim is among the players, she occupies the first position.

and

$$\begin{aligned} v(1) &= v(1,3) = d_1 \\ v(2) &= v(3) = v(2,3) = 0 \\ v(1,2) &= d_1 + d_2 \\ v(1,2,3) &= d_1 + d_2 + d_3 = D \end{aligned}$$

Simultaneous and sequential liability games satisfy weak superadditivity.

The *marginal contribution* of each player to coalitions to which he belongs is a central concept in allocation theory: for any given  $S \subset N$ , the marginal contribution of player  $i \in S$  to coalition  $S$  is defined by  $Cm_i(S) = v(S) - v(S \setminus i)$ . In the framework of our model,  $Cm_i(S)$  is the marginal damage of injurer  $i$  to coalition  $S$ . It measures the additional damage caused by injurer  $i$ , with reference to the potential damage of coalition  $S$ . For instance,  $d_2$  is the marginal damage of player 2 to coalition  $\{1,2\}$  while  $D = d_1 + d_2 + d_3$  is the marginal damage of player 1 to coalition  $\{1,2,3\}$ .

Two players are said to be *equal* if they contribute equally to all coalitions to which they both belong: they are interchangeable in terms of position. In the simultaneous case, all players are equal. In the sequential case, two players are equal if (and only if) they are consecutive and the first causes no immediate damage. For instance, players 1 and 2 are equal *if and only if*  $d_1 = 0$ .

### 2.3 The core

The core of a game  $(N, v)$  is a concept introduced by Gillies (1953). It is the set of allocations that give to all coalitions *at least* what they are worth:

$$C(N, v) = \left\{ x \in (x_1, \dots, x_n) \left| \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \text{ for all } S \subset N \right. \right\}$$

i.e. no coalition can formulate an objection against core allocations on the basis of its worth.<sup>16</sup>

Applied to a 3-player game, it is the set of allocations  $(x_1, x_2, x_3)$  such that  $x_i \geq v(i)$  for all  $i$ , and  $x_i + x_j \geq v(i, j)$  for all  $i \neq j$ . Equivalently, it is the set of allocations  $(x_1, x_2, x_3)$  such that

$$\begin{aligned} v(1) &\leq x_1 \leq v(N) - v(2,3) \\ v(2) &\leq x_2 \leq v(N) - v(1,3) \\ v(3) &\leq x_3 \leq v(N) - v(1,2) \end{aligned}$$

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<sup>16</sup> The core may well be empty: weak superadditivity is a necessary but not sufficient condition for non-emptiness of the core.

The left-hand sides are the potential damage of the individual players and the right-hand sides are their additional damage. Hence, the core is precisely the set of *unobjectionable adjudications* as defined in the introduction: each player pays *at least* his potential damage and *at most* his additional damage.<sup>17</sup>

In the simultaneous case, the core imposes no restriction:  $0 \leq x_i \leq D$  for all  $i \in N$ . In the 3-player sequential case, the core is the set of allocations  $(x_1, x_2, x_3)$  such that:

$$d_1 \leq x_1 \leq d_1 + d_2 + d_3$$

$$0 \leq x_2 \leq d_2 + d_3$$

$$0 \leq x_3 \leq d_3$$

Hence, the core of simultaneous and sequential liability games is always nonempty. We observe that unobjectionable adjudications satisfy a basic fairness principle: no one covers a damage that has occurred "*upstream*" in the sequence. As a result, the first player has always to cover the initial damage.

## 2.4 The symmetric Shapley value

For a given game  $(N, v)$ , the Shapley value is an *allocation rule* that specifies for each player a share in  $v(N)$ . It is defined as a weighted average of his marginal contributions:

$$SV_i(N, v) = \sum_{S \subset N} \alpha_n(s) (v(S) - v(S \setminus i)) \quad i = 1, \dots, n \quad (2)$$

where the weights only depend on coalition size.<sup>18</sup> They are given by:

$$\alpha_n(s) = \frac{(s-1)!(n-s)!}{n!}$$

As such, it is just a formula but it can be axiomatized. There exist several characterizations in the literature beyond Shapley's original one.<sup>19</sup> We retain here the alternative axiomatization due to Young (1985) because it is more appropriate within our context. Young proves that it is the *unique* allocation rule that satisfies the following properties:

### *Symmetry*

Equal players are entitled to equal shares.

### *Monotonicity*

If a game  $(N, v)$  is modified in such a way that the marginal contributions of a player do not decrease, then the amount allocated to that player cannot decrease.

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<sup>17</sup> This is true as well for 2-player liability games.

<sup>18</sup> We use lower case letter to identify the size of a set:  $s$  is the cardinal of  $S$ .

<sup>19</sup> See for instance Moulin (1988) or Winter (1994).

Symmetry is nothing but *equal treatment of equals*. Monotonicity is a strong independence axiom: it requires that what is allocated to a player only depends on his marginal contributions, independently of the way the other players contribute. *Efficiency* is a third axiom that is actually included in the definition of an allocation rule: the value of the game  $v(N)$  is *exactly* distributed.

Applied to a simultaneous liability game, no need for hard computations: by symmetry, the Shapley value imposes equal division. In the sequential 2-player case, we retrieve the rule (1) with equal weights. Indeed, using (2), we obtain the following allocation:

$$\begin{aligned}x_1 &= \frac{1}{2}d_2 + d_1 \\x_2 &= \frac{1}{2}d_2\end{aligned}$$

This "triangular" formula easily extends to any number of players. In the 3-player case, we have:

$$\begin{aligned}x_1 &= \frac{1}{3}d_3 + \frac{1}{2}d_2 + d_1 \\x_2 &= \frac{1}{3}d_3 + \frac{1}{2}d_2 \\x_3 &= \frac{1}{3}d_3\end{aligned}\tag{3}$$

## 2.5 The weighted Shapley value

Removing symmetry allows equal players to be treated differently, opening the way to a family of values, called *weighted Shapley values*, obtained by assigning weights to players.<sup>20</sup> In the 2-player and 3-player cases, given non-negative weights summing to one, we obtain the following allocations:

$$\begin{aligned}x_1 &= w_1d_2 + d_1 \\x_2 &= w_2d_2 = (1 - w_1)d_2\end{aligned}\tag{4}$$

and

$$\begin{aligned}x_1 &= w_1d_3 + \frac{w_1}{w_1 + w_2}d_2 + d_1 \\x_2 &= w_2d_3 + \frac{w_2}{w_1 + w_2}d_2 \\x_3 &= w_3d_3 = (1 - w_1 - w_2)d_3\end{aligned}\tag{5}$$

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<sup>20</sup> Weighted Shapley values have been axiomatized. See for instance Kalai, and Samet (1987) or Dehez (2011).

These are again triangular formulas, with appropriate weighting. Notice that in the 3-player case, (5) is valid as long as  $w_1 + w_2 > 0$ . In the case where  $w = (0, 0, 1)$ , the last player covers his additional damage  $d_3$  but there is an *indetermination* concerning the division of  $d_2$ . A selection has to be made. Because weights are equal, the natural solution is to apply the symmetric Shapley value to the 2-player game restricted to the coalition  $\{1, 2\}$ . The corresponding allocation is then given by:

$$x_1 = \frac{1}{2}d_2 + d_1$$

$$x_2 = \frac{1}{2}d_2$$

$$x_3 = d_3$$

The allocation that imposes to the first player to cover the entire damage  $D$  corresponds to  $w = (1, 0, 0)$ . The allocation  $x = (d_1, d_2 + d_3, 0)$  corresponds to  $w = (0, 1, 0)$ : the last player is exempted and the second player covers his marginal damage  $d_2 + d_3$ . If only one player is assigned a zero weight, he is exempted, except of course for the first player who has to pay at least  $d_1$ . Hence,  $x_i = 0$  if  $w_i = 0$  for all  $i \geq 2$  and  $x_1 = d_1$  if  $w_1 = 0$ .

It is easily verified that the allocation corresponding to any non-negative weights  $w = (w_1, w_2, w_3)$  belongs to the core. On the other hand, Monderer, Samet and Shapley (1992) have shown that core allocations are weighted values. Hence, there is a one-to-one relationship between weighted adjudications and unobjectionable adjudications: a weighted adjudication is unobjectionable and, vice-versa, unobjectionable adjudications reveal weights.<sup>21</sup>

## 2.6 Beyond three players

The definition of the Shapley value, weighted or not, is easily extended to any number of players: the triangular formulas (3) and (5) indeed extend to any  $n \geq 3$ . It goes differently for the concept of unobjectionable adjudication. As mentioned in the introduction, it can be extended to accommodate more than three players by going from individual players to coalitions of players. Indeed, consider a core allocation  $x$  and a coalition  $S \subset N$ . By definition of the core, we have:

$$\sum_{i \in S} x_i + \sum_{i \in N \setminus S} x_i = v(N) \quad \text{and} \quad \sum_{i \in N \setminus S} x_i \geq v(N \setminus S)$$

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<sup>21</sup> In general, core allocations are weighted values. The opposite inclusion is only verified for the class of convex games introduced by Shapley (1971). Simultaneous and sequential liability games are convex. See Dehez, and Ferey (2013) for more details.

Combining these two conditions, we obtain:

$$\sum_{i \in S} x_i \leq v(S) - v(N \setminus S)$$

Hence, core allocations satisfy the following inequalities:

$$\sum_{i \in S} x_i \geq v(S) \quad \text{and} \quad \sum_{i \in S} x_i \leq v(S) - v(N \setminus S) \quad \text{for all } S \subset N$$

and, for any given coalition  $S$ , these inequalities are *equivalent*. Applied to liability games, this corresponds to the conditions C1 and C2: no *coalition* pays less than its potential damage or more than its additional damage. With this definition of unobjectionable adjudication, all what precedes carries over, in particular the equivalence between unobjectionable adjudications and weighted adjudications. Notice that C1 and C2 are equivalent conditions: an adjudication that verifies one, automatically verifies the other.



### 3. APPLYING THE WEIGHTED SHAPLEY VALUE TO THE LAW

Liability games mathematically defined in the previous section and their solution concepts are relevant to improve our understanding of apportionment issues. As cooperative game theory is used less in law and economics literature than non-cooperative research agenda, we further develop the scope of our approach for the law by insisting on the both the normative and the descriptive use of solution concepts. These two perspectives are related but we deal with them separately to be the clearest as possible on the findings of the model.

#### 3.1 The Shapley value as a normative tool

"Normative aspects of game theory may be sub classified using various dimensions. One is whether we are advising a single player (or group of players) on how to act best in order to maximize payoff to himself, if necessary at the expense of the other players; and the other is advising society as a whole (or group of player) of reasonable ways of dividing payoff among themselves. The axis I'm talking about has the strategist (or the lawyer) at one extreme, the arbitrator (or the judge) at the other." (Aumann 1985, p. 38). In the following, we use the term *normative* in the second sense, the one of the judge.

The Shapley value is just one solution among others. Therefore, why should the Shapley value be preferred to any other solution? Should a Court follow apportionment based on the Shapley value? Here we rely on three major arguments to answer this question. First, the properties of the Shapley value are meaningful for the law and need to be carefully examined to assess its normative content and acceptability. Second, compared to other solutions, the Shapley value seems more relevant to correctly apportion damage among injurers in legal contexts. Third, normative statement in terms of game theory has to be compared with traditional law and economics criteria, namely the minimization of social costs.

##### 3.1.1 The Shapley value properties

The symmetric Shapley value is a fair compromise between tortfeasors' concurrent claims. Two arguments need to be elaborated to see why. First, Shapley's formula is based on marginal damages. In this sense, the Shapley value is an evaluation of the degree of causation of each wrongdoing act and can be considered as a useful benchmark to evaluate whether an injurer was strongly or weakly causally involved in the damage.<sup>22</sup> Second, the axiomatic characterization of the value identifies the foundations of this allocation procedure, in particular efficiency, symmetry and monotonicity.

Efficiency imposes that the total damage be exactly paid by the tortfeasors. This property is needed for the law: damage has to be totally recovered by the victim and, at the same time, punitive damages put aside, the victim cannot get more than his damage.

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<sup>22</sup> For a similar statement from a philosophical perspective, see Braham, and van Hees (2009).

Symmetry states that two injurers with identical marginal damage to all the coalitions of which they are member should pay the same amount. Quoting Young (1994, p. 1215), "Of all properties that characterize the Shapley value, symmetry seems to be the most innocuous [...] because it calls for a judgment about what should be treated equally. [...] the symmetry axiom is not plausible when the partners [...] differ in some respect other than [benefit] that we feel has a bearing on the allocation". That is why our approach leaves open the possibility to consider some elements beyond causation, as the degree of fault and responsibility. These elements rely on distributive justice and not corrective justice.

Monotonicity states that if the marginal damages of an injurer decrease, what he is asked to pay should not increase, independently of possible changes in marginal damages of other injurers. In other terms, the share of an injurer should depend exclusively on his marginal damages.<sup>23</sup>

The axiomatic foundations of the value apply to the set of all transferable utility games. Looking at sequential liability situations, a natural question is to identify properties that produce the above triangular formula. Here are two properties that are particularly appropriate in our context.

#### *Zero immediate damage*

If an injurer causes no immediate damage, his share and the share of his successor coincide.

#### *Upstream independence*

The share of an injurer does not depend on the damage caused by the injurers that precede him.

Zero immediate damage is nothing but symmetry. We have indeed seen that when  $d_i = 0$  for some  $i < n$ , the injurer  $i$  and  $i + 1$  are interchangeable. Upstream independence says that the share of a injurer is independent of the immediate damages caused by his predecessors.

**Proposition** In a sequential liability case, there is a unique allocation rule  $\varphi$  that satisfies zero immediate damage and upstream independence. It is the Shapley value of the corresponding liability game.

Proof We look for the rules  $\varphi$  satisfying the following two properties:

$$\begin{aligned} d_i = 0 \text{ for some } i < n &\Rightarrow \varphi_i(d) = \varphi_{i+1}(d) \\ d_j = d'_j \text{ for all } j = 1, \dots, i-1 &\Rightarrow \varphi_i(d) = \varphi_i(d') \end{aligned}$$

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<sup>23</sup> Actually, that property is enough to characterize the Shapley value. See Young (1985).

Consider the case  $n = 3$ . If  $d = (0, 0, d_3)$ , efficiency and zero immediate damage imply:

$$\varphi_1(0, 0, d_3) = \varphi_2(0, 0, d_3) = \varphi_3(0, 0, d_3) = \frac{d_3}{3}$$

Upstream independence then ensures that  $\varphi_3(d) = d_3/3$  for all  $d \in \mathbb{R}_+^3$ . Hence, if  $d = (0, d_2, d_3)$ , efficiency and zero immediate damage imply:

$$\varphi_1(0, d_2, d_3) = \varphi_2(0, d_2, d_3) = \frac{1}{2}(d_2 + \frac{2}{3}d_3) = \frac{1}{2}d_2 + \frac{1}{3}d_3$$

Upstream independence then ensures that  $\varphi_2(d) = d_2/2 + d_3/3$  for all  $d \in \mathbb{R}_+^3$ . We then are left with  $d_1$  that must then be paid by the first player:

$$\varphi_1(d) = d_1 + \frac{1}{2}d_2 + \frac{1}{3}d_3 \quad \text{for all } d \in \mathbb{R}_+^3$$

Hence, zero immediate damage and upstream independence (together with efficiency) define a unique rule and it coincides with the Shapley value of the associated liability game. The argument extends to any number of players, starting from the last player and proceeding backward. ■

### 3.1.2 Comparing the Shapley value with alternative allocation rules

The reason why we have insisted on the Shapley value as a useful guide for the Court is also due to the advantages of the Shapley value compared to other allocation rules, taking into account the context. We will consider three alternative apportionment schemes: egalitarian rule, marginal damage and nucleolus.

The egalitarian rule is the simplest allocation. It imposes equal division: each injurer pays the same amount,  $x_i = d/n$  for all  $i = 1, \dots, n$ . Such an apportionment is not necessarily fair insofar as it does not take account of the relative involvements of the player in the occurrence of the damage. We have seen that it applies when damage would not have occurred if one of the injurer had not been present.

An alternative could be to impose to players to pay only for their marginal contribution to the *total* damage  $Cm_i(N) = v(N) - v(N \setminus i)$ . A correction is then needed to ensure efficiency, resulting in the following allocation rule:

$$x_i = Cm_i(N) + \frac{1}{n} \left( v(N) - \sum_{j \in N} Cm_j(N) \right) \quad \text{for all } i = 1, \dots, n$$

It coincides with the Shapley value in the 2-player case but is much different when more than two players are involved. In the 3-player case, we obtain:

$$x_1 = \frac{1}{3}d_3 + \frac{2}{3}d_2 + d_1$$

$$x_2 = \frac{1}{3}d_3 + \frac{2}{3}d_2$$

$$x_3 = \frac{1}{3}d_3 - \frac{1}{3}d_2$$

Applied to sequential liability games, this rule does not satisfy upstream independence and also fails to satisfy monotonicity: the immediate damage caused by the second injurer affects what the last injurer is asked to pay. In addition, it is affected negatively: a decrease in  $d_2$  leads to an increase in  $x_3$ .

The *nucleolus* is an allocation rule introduced by Schmeidler (1969). It builds on the notion of *least core* that is concerned with the minimization of the differences  $e(x, S) = v(S) - x(S)$  between what coalitions are worth and what they receive: if positive, the "excess"  $e(x, S)$  measures the loss (or gain if negative) for coalition  $S$  if its members accept  $x$  instead of forming their coalition. Hence, an allocation  $x$  belongs to the core if no coalition would gain by forming:  $e(x, S) \leq 0$  for all  $S \subset N$ . The least core is the set of allocations that minimizes the largest excess, a concept introduced by Maschler, Peleg and Shapley (1979). The nucleolus goes further by comparing the largest excesses *lexicographically* so as to eventually retain a unique allocation.<sup>24</sup> If the core is nonempty, the nucleolus defines an allocations located centrally within the core. It is the allocation that "minimizes dissatisfaction, with priority to the coalitions that are most dissatisfied", to quote Shubik (1982, p. 339). The nucleolus satisfies efficiency and symmetry but not monotonicity. In the 3-player sequential case, the nucleolus has two parts, depending on the relative values of  $d_2$  and  $d_3$ . If  $d_3 \leq 2d_2$ , it produces the following allocation:

$$x_1 = \frac{1}{4}d_3 + \frac{1}{2}d_2 + d_1$$

$$x_2 = \frac{1}{4}d_3 + \frac{1}{2}d_2$$

$$x_3 = \frac{1}{2}d_3$$

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<sup>24</sup> This corresponds to the *leximin* criterion proposed by Rawls in his *Theory of Justice* (1971).

If instead,  $d_3 \geq 2d_2$ , we have:

$$x_1 = \frac{1}{3}d_3 + \frac{1}{3}d_2 + d_1$$

$$x_2 = \frac{1}{3}d_3 + \frac{1}{3}d_2$$

$$x_3 = \frac{1}{3}d_3 + \frac{1}{3}d_2$$

Clearly the nucleolus induces a rule that fails to satisfy monotonicity and upstream independence.

The following table gives an overview of the different allocation rules studied and their properties, confirming the pertinence of the Shapley value. It of course results from Young's characterization of the Shapley value and from our characterization on the set of sequential liability games.

	Equal division	Marginal damage	Nucleolus	Shapley value
Symmetry	Yes	Yes	Yes	Yes
Monotonicity	No	No	No	Yes
Upstream independence	No	No	No	Yes

### 3.1.3. Incentives and the Shapley value

Even if our approach does not directly deal with the incentive aspect of apportionment, it leaves a room open for further development making a bridge between cooperative and non-cooperative approaches on multiple causation issues. As we asserted in introduction, most of the literature in law and economics has developed an *ex ante* perspective: the main issue of apportionment is to provide efficient incentives for future injurers. From a normative perspective, it is needed to consider the relationships between the Shapley value solutions and the efficiency criteria used in law and economics literature, i.e. the minimization of the social costs.

One of the results of the non-cooperative literature in law and economics about apportionment is that, under certain circumstances, minimization of social costs requires multiple injurers to pay together more (or in some cases less) than the total amount of damage.<sup>25</sup> In the simplest two player's case, minimization of social costs may require the first

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<sup>25</sup> See Faure et al. (2007, p. 123).

tortfeasor to pay  $d_1 + d_2$  and the second tortfeasor to pay  $d_2$ . As such, the total amount of the offsettings paid by tortfeasors would be  $d_1 + 2d_2$  leading to an overcompensation of the victim. Avoiding overcompensation would be possible by decoupling compensation and damages paid. Even if decoupled liability designs existed it could be considered as unfair since causation requirements would be violated and tortfeasors would pay more than what they have actually caused. As the Shapley value respects the efficiency axiom and reduces the size of the set of acceptable allocations, choosing a weighted Shapley value for apportioning the damage among tortfeasors does not necessarily lead to an optimal (*ex-ante*) incentives scheme. We face a trade-off between minimization of the social costs and fairness principles.

However, one step further could be proposed to fill this gap. As the different allocations belonging to the core – which are the weighted Shapley-values – lead to different incentives schemes on tortfeasors, the minimization of social cost criteria could be used to choose among them. In other words, it would be acceptable to choose, within the core, the allocation which provides the best *ex ante* incentives in terms of minimization of social costs. This is a second best argument on which it could be possible to elaborate further to build a bridge between the *ex post* approach and the *ex-ante* approach of causation.

### 3.2 The Shapley value as a descriptive tool

The solution concepts in cooperative game theory have not only to be understood as normative tools to guide a Court. They also provide a framework to better describe existing norms and Courts' decisions.<sup>26</sup> We now address the descriptive interpretation of the solution concepts to correctly describe Courts' practices. First, we analyze some famous cases to show that adjudications may be rationalized in term of the Shapley value. We then develop further the argument by providing a rationalization of the principles and methods advocated by *the Restatement* to apportion damage among multiple tortfeasors – the "two-steps" method – in terms of the Shapley value.

#### 3.2.1 Example and cases

The most illustrative examples of our model are the successive accident cases where the tortfeasors tortious acts are related.<sup>27</sup> To illustrate our approach, we first study in details a particular case. In *Webb v Barclays Bank Plc & Anor*, the England and Wales Court of

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<sup>26</sup> "The distinction between the descriptive and the normative modes is not as sharp as might appear, and often it is difficult to decide which of these two we are talking about. For example, when we use game or economic theory to analyze existing norms (e.g. law), is that descriptive or is it normative? We must also be aware that a given solution concept will often have both descriptive and normative interpretations..." (Aumann 1985, p.37). In a famous paper, Aumann, and Maschler (1985) have contributed to this view by providing a game theoretic analysis of a bankruptcy problem from the Talmud.

<sup>27</sup> We exclude unrelated cases insofar as the second tortious act is not a legal cause of the damages up: apportionment is simple and is proportionate to each harm separately evaluated.

Appeal (Civil Division) had to solve a multiple and successive causation cases.<sup>28</sup> Here are the facts. Mrs. Webb contracted polio in the second year of her life and stayed with leg and knee vulnerability. In 1994, she was employed by Barclays Bank (thereafter the Bank) and stumbled and fell in their forecourt. She suffered pain and was then cured by the Portsmouth Hospital Trust (thereafter the Trust). After several medical treatments, the Trust advised Mrs. Webb to get an amputation above the knee. She accepted. A few months later, an independent report from others doctors shows that the Trust was negligent about advises provided to Ms. Webb and that such a medical operation required a more complete examination. Mrs. Webb decided to claim against the Bank and the Trust. In 2000, the Bank settled with Ms. Webb for the entirety of the damage (£. 165,953). The Bank then had a recovery claim against the Trust to recover his share back. Apportionment and the recovery claims from the Bank to the Trust were disputed.

First the Court wonders whether "when an employee is injured in the service, and by the negligence, of her employer, his liability to her is terminated by the intervening negligence of a doctor brought in to treat the original injury, but who in fact made it worse." (§52). "The answer to this first issue is negative and the negligence in advising amputation did not eclipse the original wrong-doing. The Bank remains responsible for its share of the amputation damages. The negligence of [the Trust] was not an intervening act breaking the chain of causation." (§57). Therefore, the entire damage has to be apportioned among the two injurers.

Second, the Court addresses the issue of apportionment. The logic of apportionment provided by the Court is exactly the same as a weighted Shapley value. The Court begins by dividing the final damage in two part, basis A and basis B: "First, (Basis A) there was the tripping accident, brought against the claimant's employers, the Bank, for their negligent failure to maintain their forecourt. [...] Second (Basis B), there was the claim for the doctor's negligent advice, as a result of which the leg was amputated." (§ 46). Basis A – which is  $v(1)$  in terms of our model – is evaluated at £. 53,945 and Basis B – which is  $v(12)$  in terms of our model – at £. 112,008. Then, the Court assesses the degree of responsibility: "The Bank, by their negligent maintenance of the forecourt, was responsible for getting the vulnerable Mrs. Webb before the doctors employed by the Trust. But it was the latter's' negligence that was much more responsible for the amputation and all that went with it. In all the circumstances, we assess the Bank's responsibility at 25% and the Trust's at 75%". (§ 59) The final apportionment ordered by the Court is therefore for the Bank: Basis A plus 25% of Basis B and for the Trust 75% of Basis B. In terms of our model, the allocation chosen by the Court is the weighted Shapley value as defined by equation (4) with  $w_1 = 0.25$  and  $w_2 = 0.75$ .

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<sup>28</sup> Webb v Barclays Bank Plc & Anor (2001) EWCA Civ 1141 (16 July 2001).

Many others cases and litigations are covered by our model: enhanced injury, background conditions, victim's contribution and also some nuisances or product liability cases. In these issues, a common mathematical structure can be identified once the different tortfeasors (including the victim) follow a temporal chain of causality: in these cases, tortious acts of the tortfeasor  $i$  are a physical cause of "direct" damage  $d_i$  and a proximate or legal cause of the aggravated damages up the liability sequence (the enhanced injuries  $d_j$  with  $j > i$ ). We provide further examples of these different kinds of litigation. Thereafter, all these cases will be named "successive injury cases".

*Successive accident cases.* In *Maddux*, the first tortfeasor hits the plaintiff's car and thirty second later, a second driver hits the car and caused other injury. The causal events are so close that the chain of injuries may be considered as a single case.<sup>29</sup>

*Background conditions.* In *Steinhauser*, the Court had to adjudicate a case where the tortious act of the defendant had caused a "chronic schizophrenic reaction" from the plaintiff.<sup>30</sup> The Court held that the defendants could explore the possibility of plaintiff having developed schizophrenia regardless of the accident.

*Victim's contribution.* In *Prospectus Alpha Navigation Co*, the plaintiff's ship was tied up at the defendant's dock.<sup>31</sup> Because of a negligent tortious act of the plaintiff's crew, the ship caught fire. But the defendant was also negligent: he send the plaintiff's ship away before the fire being completely extinguished. Then, the fire caused further damage. In *Dillon*, a young boy was on a high beam of a bridge trestle.<sup>32</sup> He lost his balance and was falling to the rocks when he grabbed the electric wires, negligently exposed by the defendant, which killed him.

*Product liability.* In *Hillrichs*, the Court considered that a jury could evaluate the extent of the enhanced injury.<sup>33</sup> A corn harvesting machine was not equipped with an emergency stop device and the plaintiff lost his fingers after his hand had been entangled in. The Court considered that some evidence showed that the injury would have been different with a stop emergency device. In *Reed*, the plaintiff's was involved in a car accident in which the shattering of the fiberglass top of his car hurts his arm.<sup>34</sup> The expert testified that such injury would have been avoided by a metal top. The Court considered that estimation of the enhanced damages was possible.

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<sup>29</sup> *Maddux v. Donaldson*, 108 N.W.2d 33 (Mich. 1961).

<sup>30</sup> *Steinhauser v. Hertz Corp.*, 421 F.2d 1169 (2d Cir.1970). See also *Lancaster v. Norfolk and Western railway Co.*, 773 F. 2d 807, 822 (7th Circuit. 1985).

<sup>31</sup> *Prospectus Alpha Navigation Co v. North Pacific Grain Growers, Inc.*, 767 F. 2d 1379 (9th Cir. 1986).

<sup>32</sup> *Dillon v. Twin State Gas & Electric Co.*, 163 Atl. 111 (N.H.1932).

<sup>33</sup> *Hillrichs v. Avco Corp.*, 478 N.W.2d 70 (Iowa 1991).

<sup>34</sup> *Reed v. Chrysler Corp.*, 494 N.W.2d 224 (Iowa 1992).



### 3.2.2 A more systematic view: The Restatement and the Shapley value

The usefulness of the Shapley value to better understand apportionment principles in law may be systematized. One of the innovation proposed by the *Third Restatement* compared to the *First* or the *Second* is a "two-step procedure" to apportion damage among tortfeasors.<sup>35</sup> The method provides a unified framework taking account the different issues: causation, degree of responsibility, divisibility, inconsistent verdicts *etc.* The method is stated as follow: "The factfinder divides divisible damages into their indivisible component parts. The factfinder then apportions liability for each indivisible component part under Topics 1-4. For each indivisible component part, the factfinder assigns a percentage of comparative responsibility to each party or other relevant person [...]. The percentages of comparative responsibility for each component part add to 100 percent [...]. The plaintiff is entitled to judgment in an amount that aggregates the judgments for each component part".<sup>36</sup> This method corresponds to the weighted Shapley value.

First, the *Restatement* states that the damage must be divided by causation when it is possible to assign to one tortfeasor or to subsets of tortfeasors the part of the damage this subset has caused alone.<sup>37</sup> The characteristic function of a liability game provides such a division of damage. Reciprocally, the factfinder or the jury instructed by a Court to divide the damage seeking to assign to each subset of tortfeasors the damage they would have caused alone, defines a characteristic function.

Sometimes, the task is easy because the aggravated damages  $d_i$  are perfectly observable; sometimes, a counterfactual is needed. The factfinder wonders which amount of damage would have occurred if one or several tortfeasors had not acted tortuously and defines potential damages. Our model captures these features, given that all the coalitions but the grand coalition are only hypothetical.<sup>38</sup> For example, in *Dillon*, Court has used potential damages to drastically reduce the amount paid by the electric company by holding that even if the company had not been negligent, the boy would have suffered important damage due to his fall.<sup>39</sup> The only damage the electric company has caused is, at most, the difference between actual damage and potential one. In other words, the Court has divided the harm by evaluating the potential amount of damage due to the fall *alone*.<sup>40</sup> Similar legal reasoning

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<sup>35</sup> Topic 5 of the *Restatement* is entitled "Apportionment Of Liability When Damages Can Be Divided By Causation". See also the *Restatement* (second) § 879 and Boston (1996).

<sup>36</sup> The *Restatement*, Topic 5, §26, *comment c*.

<sup>37</sup> Interestingly enough, the *Restatement* mentions explicitly the "set" of tortfeasors: "Divisible damages may occur when a part of the damages was caused by one set of persons in an initial accident and was then later enhanced by a different set of persons" (the *Restatement*, Topic 5, *Reporters' note, comment f*).

<sup>38</sup> We rely on the classical distinction between prospective causation and retrospective one, see note 8 *supra*.

<sup>39</sup> *Dillon v. Twin State Gas & Electric Co.*, 163 Atl. 111 (N.H.1932).

<sup>40</sup> Obviously, if the boy had not lost his balance, the tortious act would have not been damageable. On the contrary, if the electric company had not been negligent, a less important damage would have occurred. The key-element the factfinder has to know is whether the boy had already lost his balance before grabbing the

could be found in other issues.<sup>41</sup> Once defined the characteristic function, the question to know how to divide divisible and indivisible parts among tortfeasors still holds. We now discuss this point.

Once damage is divided, the first step of the methodology provided by the *Restatement* is to apportion damage by causation, namely: each tortfeasor should pay at least for the damage he would have caused alone and at most for the additional damage he has caused.<sup>42</sup> For most legal theorists, it would be unfair for a tortfeasor to pay for more than what he has caused. This basic principle inspired by corrective justice is accepted as the cornerstone of all acceptable apportionment rules. As asserted by Robertson (2009, p.1008), following Carpenter, "it has long been regarded as a truism that 'a defendant should never be held liable to a plaintiff for a loss where it appears that his wrong did not contribute to it, and no policy or moral consideration can be strong enough to warrant the imposition of liability in such [a] case'." As soon as the sum of the payments due by each tortfeasor exactly covers the harm suffered by the plaintiff<sup>43</sup>, the core of a 3-player liability game is the subset of allocations that verify two conditions ("*non-objectionable adjudications*"). The first one is that the contribution of any subset of tortfeasors should be at least equal to the damage they would have caused without the intervention of the others, the second one is that no group of tortfeasors should pay more than what it has caused. Law and legal doctrine acknowledge the importance of these two restrictions to consistently apportion liability. Saying that no tortfeasor should pay more than he has caused is a legal translation of the condition C2 in our game.<sup>44</sup> Legal principles and economic conditions converge.<sup>45</sup>

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electric wires or not. Commenting this case, Chief Justice Peaslee said that "serious injury, if not dead, was certain to ensue, when he was caught upon the defendant's wires and electrocuted" and therefore, it was fair and logical that the Court allows a compensation for only such a sum "as his prospects for life and health were worth at the time the defendant's fault became causal." (Peaslee 1934, p.1134-1135).

<sup>41</sup> *Douglas Burt & Buchanan Co. V. Texas & Pacific Railway Co.*, 1922, 150 La 1038, 91 So. 503.

<sup>42</sup> This principle is one of the cornerstones of the *Restatement*: "no party should be liable for harm it did not cause" (*Restatement*, Topic 5, §26, *comment a*, see also *comments h* and *j*). That is why a one step procedure is unfair: "a court may decide to use a one-step procedure of apportionment. The factfinder determines the total recoverable damages and then assigns percentages of responsibility to each person who caused some of the damages [...]. A problem with a one-step procedure is that it may result in a party being held liable for more damages than the party caused. See *comment d*. A party's comparative responsibility is distinct from the magnitude of the injury the party caused" (*Restatement*, Topic 5, §26, *comment j*).

<sup>43</sup> A plaintiff's total aggregate recovery from all the contributing tortfeasors can never exceed the amount of his actual damages. See *Miller v. Union Pacific R. Co.*, 290 U.S. 227, 236 (1933). We do not deal with punitive damages and we consider that Courts are able to calculate the full amount of damage to be paid to the victim. *A priori*, our argument does not depend on the methods actually used by Courts to calculate damages except if the calculation leads to non-monotonicity: it could be the case, for example, when a first tortfeasor causes a disease to the victim, following by a second tortfeasor who causes death and compensation for death be less important than compensation for disease. Offsetting benefits are therefore excluded, see Porat, and Posner (2014).

<sup>44</sup> For example, in *Ravo v. Rogatnick*, the Court states that, in case of successive injuries due to medical malpractice, "the initial tortfeasor may well be liable to the plaintiff for the entire damage proximately resulting from his own wrongful acts. The successive tortfeasor, however, is liable only for the separate injury or the aggravation his conduct has caused" *Ravo v. Rogatnick*, 514 N.E.2d 1104 (N.Y. 1987). Or see *Pridham v. Cash & Carry Bldg. Center, Inc.*, 359 A.2d 193 (N.H. 1976), *Prospectus Alpha Navigation Co v. North Pacific Grain Growers, Inc.*, 767 F. 2d 1379 (9th Cir. 1986).

However, most of time, apportionment by causation is insufficient to provide a unique apportionment of the damage (in mathematical terms, the core typically contains many allocations). One remaining issue is precisely to know how to divide the indivisible components and therefore to choose one apportionment among the core-allocations. The second principle proposed by the two-step method – the apportionment by responsibility – is needed:<sup>46</sup> "the court should divide damages by causation and then, *for each component part*, apportion liability by shares of responsibility." Fault degrees of each tortfeasor are introduced and play the role of relative weights. Dividing indivisible damage by responsibility in the sense of the *Restatement* consists in assigning weights to each tortfeasor in order to divide the indivisible components. Judge could consider arguments which justify treating unequally the tortfeasors, for example, because their degrees of fault are different. It is easy to show that some examples provided by the *Restatement* follow a weighted Shapley value logic.<sup>47</sup>

Weighted Shapley-value offers possible compromises consistent with conditions C1 and C2 and with the evaluation by the judge of the responsibility of each. And the weighted Shapley value mathematically distinguishes between causation and responsibility apportionments. Reciprocally, each core-allocation is a particular weighted Shapley-value. As

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<sup>45</sup> See the *Restatement (second) of Torts*: "it should be noted that there are situations in which the earlier wrongdoer may be liable for the entire damage, while the later one will not. Thus an original tortfeasor may be liable not only for the harm which he has himself inflicted, but also for the additional damages resulting from the negligent treatment of the injury by a physician. The physician, on the other hand, has played no part in causing the original injury, and will be liable only for the additional harm caused by his own negligence in treatment." (*Restatement (second) of Torts*, 16.1.A., §433A, *comment c*; see Keaton et al. (1984, p. 352).

<sup>46</sup> Emphasis added. See the *Restatement* Topic 5, §26, *Reporters' note, comment d*. This comment criticizes *Alpha navigation* because the additional damage is partly due by the first tortfeasor insofar as without his tortious act the latter damage would have not occurred: "The court stopped with causal division by holding that the defendant was liable for *all* the damage caused by its decision to send the ship away. That is not consistent with the goals of comparative responsibility. The plaintiff's negligence also caused the extra damage; but for the original fire, there would have been no damage".

<sup>47</sup> See the *Restatement*, Topic 5, §26, *Reporters' note, comment c*: Let's study one of the examples provided by the *Restatement* to illustrate the two-step procedure: "Consider a case in which D, the driver of an automobile, is alleged to have negligently driven an automobile into a highway guardrail. An alleged defect in the automobile's door latch causes the passenger door to open and P, the passenger, to be ejected. P suffers serious neurological injuries and sues D and M, the automobile's manufacture [...]. The court instructs the jury that it must find what damages P would have suffered if the door had not opened (assuming the jurisdiction recognizes that hypothetical injury as a cognizable injury for purposes of causal division) [...]. After making that determination, the jury decides if D and M are legally responsible and assigns percentages of responsibility to them [...]. If the jury found that P would have suffered some damages if the door had remained closed, damages are divisible. The jury determines if D was negligent, M's automobile was defective, the negligence caused the entire damages, and the defect caused the enhanced injury. It finds the amount of damages for the enhanced injury and the damages for the entire injury. The jury then assigns percentages of responsibility to D and M for the enhanced injury". In terms of our model, the court determines the set of tortfeasors  $N = \{D, M\}$  and instruct the jury to determine  $d_1$  and  $d_2$  (damage division). This defines the characteristic function  $v$  associated to the set of tortfeasors  $\{D, M\}$ :  $v(D) = d_1$ ,  $v(D, M) = d_1 + d_2$  and  $v(M) = 0$ . The legal issue is to solve the transferable utility game  $(N, v)$ : the initial damage is entirely paid by D since he is the only cause of this part of the damage and the enhanced injury is shared between the two tortfeasors by assigning to each of them a degree of responsibility  $w_D$  et  $w_M$  knowing that the sum equals 100%. Then the payment due by each tortfeasor from this two-step procedure is exactly the weighted Shapley value associated to the game  $(N, v)$ .

such, it is possible to consider that, as soon as a Court chooses an unobjectionable adjudication, it reveals the degree of responsibility of each tortfeasor.

## 4. CONCLUDING REMARKS

To conclude, we develop further comments and propose possible extensions beyond the sequential liability models.

First, the model covers a wide range of cases and provides a better "comprehension" of them. By comprehension, we mean that our model defines as a class of games covering a large variety of cases and identifies the common structure that lies behind them. In other words, it identifies a same mathematical structure unifying all sequential liability litigations. It is therefore interesting enough to know whether Courts use a same rule to apportion damage among tortfeasors in the cases belonging to a same class of games. The aim of the *Restatement* is precisely to provide such a general method and we have shown this method is deeply justified in terms of rationality as soon as it appears as the implementation as a weighted Shapley value.

Second, one of the main implications of our findings is the relationships between axiomatic reasoning, rationality and legal adjudication: by using an axiomatic method to determine the shares paid by each tortfeasor and by characterizing different solutions in terms of axioms, the discussion about the best way to apportion damage among tortfeasors is improved in terms of impartiality and rationality. One step further would be to determine the incentive effects of the implementation of a Shapley value to make clearer the trade-off between fairness and minimization of social costs.

Third, and more importantly, it is possible to extend our approach to cover other types of multiple causation cases, leading to different liability games. That requires the understanding of the structure of the multiple causation at stake. For instance, one possible extension deals with over determination cases or preemptive causation that lead to paradoxical conclusions in legal theory. Consider the famous example of two fires that jointly destroy a house. A strict "*but for test*" would lead to consider that none of the fire is a cause since the damage would have occurred anyway. Tortfeasors have already argued that they have no obligation to compensate the victim insofar as the causal link is missing. Referring to potential damages, the characteristic function is given by  $v(12) = v(1) = v(2) = D$ . This game differs from the unanimity game and is not even weakly superadditive. It admits no core allocation i.e. there exists no unobjectionable adjudication. However, the symmetric Shapley value is well defined: the players being interchangeable, it produces the equal division  $(D/2, D/2)$ .

Regarding information, as our approach is based on *ex post* causation, coalitions have to be understood as counterfactual states of the world (the state of the world which would have occurred, all things being equal, if one agent had not tortuously acted). In the sequential liability game, this task is simple and actually requires little information (only  $n$  numbers, the  $d_i$ , which often are perfectly observable). In other cases, it could be difficult to precisely identify the counterfactual states of the world. Take for instance the asbestos cases. Such a

litigation leads to apportionment issues either among several firms which have exposed the victim to asbestos products or among different insurance companies which have covered the risk for a single injurer at different periods of time. Several apportionment principals have been proposed.<sup>48</sup> As the sequential liability game, asbestos cases have a temporal structure because the disease is due to cumulated past exposure. However, asbestos cases do not share the sequential feature of our model insofar as removing an injurer  $i$  from the causality sequence does not prevent the injurers down in the sequence from increasing the expected damage, i.e. the final risk of disease. Therefore, once Courts have considered these injurers are together the cause of the disease, assigning to each coalition its value is more difficult and requires information on the risk level created by each one of the coalitions. One way could be to use the epidemiologic models describing the relationships between probability of disease and length of exposure in order to have an idea of the counterfactual states of the world. The best proxies for the counterfactual here would be the expected damages of each coalition.

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<sup>48</sup> See *Owens-Illinois, Inc. v. United Insurance Co*, 138 N.J. 437 (1994) "because multiple policies of insurance are triggered under the continuous-trigger theory, it becomes necessary to determine the extent to which each triggered policy shall provide indemnity [...]" (title VII). Court then discusses different rules that could be used to apportion the responsibility between firms and/or insurance companies.

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